Fill in the following identities. **SCORE: _____ / 14 PTS**

[a] HALF ANGLE IDENTITY:

$\sin\frac{1}{2}x = \pm\sqrt{\frac{1-\cos x}{2}}$

[d] POWER REDUCING IDENTITY:

$\cos^2 x = \frac{1+\cos 2x}{2}$

[c] NEGATIVE ANGLE IDENTITY:

$\sec(-x) = \sec x$

[d] PYTHAGOREAN IDENTITY:

$\cot^2 x = \csc^2 x - 1$

[e] DIFFERENCE OF ANGLES IDENTITY:

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

[f] SUM OF ANGLES IDENTITY:

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

[g] DOUBLE ANGLE IDENTITY: $\boxed{\text{WRITE ALL 3 VERSIONS}}$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

If $\sin t = -\frac{\sqrt{11}}{6}$ and $\pi < t < \frac{3\pi}{2}$, find the values of the following expressions. **SCORE: _____ / 30 PTS**

Write each final answer as a single fraction in simplest form, including rationalizing the denominator.

[a] $\tan 2t$

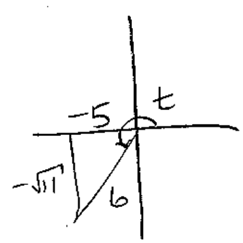
$$= \frac{2\tan t}{1-\tan^2 t}$$
$$= \frac{2\left(\frac{\sqrt{11}}{5}\right)}{1-\left(\frac{\sqrt{11}}{5}\right)^2}$$
$$= \frac{\frac{2\sqrt{11}}{5}}{\frac{14}{25}} = \frac{5\sqrt{11}}{7}$$

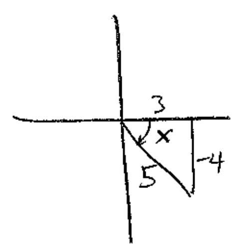
[b] $\tan\frac{1}{2}t$

$$= \frac{\sin t}{1+\cos t}$$
$$= \frac{-\frac{\sqrt{11}}{6}}{1+\left(-\frac{5}{6}\right)} \cdot \frac{6}{6}$$
$$= -\sqrt{11}$$

[c] $\sin(\underbrace{\arctan(-\tfrac{4}{3})}_{x} - t)$

$$= \sin x \cos t - \cos x \sin t$$
$$= -\frac{4}{5} \cdot -\frac{5}{6} - \frac{3}{5} \cdot -\frac{\sqrt{11}}{6}$$
$$= \frac{20+3\sqrt{11}}{30}$$





Solve the equation $2-5\cos\frac{1}{7}x=3(1-\cos\frac{1}{7}x)$. **SCORE: _____ / 14 PTS**

$$2-5\cos\tfrac{1}{7}x = 3-3\cos\tfrac{1}{7}x$$
$$-2\cos\tfrac{1}{7}x = 1$$
$$\cos\tfrac{1}{7}x = -\tfrac{1}{2}$$
$$\tfrac{1}{7}x = \frac{2\pi}{3}+2n\pi \text{ or } \frac{4\pi}{3}+2n\pi$$
$$x = \frac{14\pi}{3}+14n\pi \text{ or } \frac{28\pi}{3}+14n\pi$$

Prove the identity $\sec(-t)-\cos(-t)-\csc(-t)+\sin(-t)+\sin t\tan(-t)=\cos t\cot t$. **SCORE: _____ / 14 PTS**

$$= \sec t - \cos t + \csc t - \sin t - \sin t \tan t$$
$$= \frac{1}{\cos t} - \cos t + \frac{1}{\sin t} - \sin t - \sin t \frac{\sin t}{\cos t}$$
$$= \frac{1-\cos^2 t}{\cos t} + \frac{1-\sin^2 t}{\sin t} - \frac{\sin^2 t}{\cos t}$$
$$= \cancel{\frac{\sin^2 t}{\cos t}} + \frac{\cos^2 t}{\sin t} - \cancel{\frac{\sin^2 t}{\cos t}}$$
$$= \cos t \frac{\cos t}{\sin t} = \cos t \cot t$$

Rewrite $\cos^4 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE: _____ / 14 PTS**
Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$\cos^4 x = (\cos^2 x)^2$$
$$= \left(\frac{1+\cos 2x}{2}\right)^2$$
$$= \frac{1+2\cos 2x+\cos^2 2x}{4}$$
$$= \frac{1+2\cos 2x+\frac{1+\cos 4x}{2}}{4}\cdot\frac{2}{2}$$
$$= \frac{2+4\cos 2x+1+\cos 4x}{8}$$
$$= \frac{3+4\cos 2x+\cos 4x}{8}$$

Solve the equation $3\cos 2x + 7 = 7(1 - \sin x)$ algebraically. **SCORE: _____ / 14 PTS**

$$3(1-2\sin^2 x)+7 = 7-7\sin x$$

$$0 = 6\sin^2 x - 7\sin x - 3$$

$$= (3\sin x+1)(2\sin x-3)$$

$$\sin x = -\tfrac{1}{3} \quad \text{OR} \quad \cancel{\tfrac{3}{2}}$$

$$\text{REF ANGLE} = \sin^{-1}\tfrac{1}{3} \approx 0.3398$$

x IN Q_3, Q_4

$$x = \pi + 0.3398 + 2n\pi \approx 3.4814 + 2n\pi$$

OR

$$x = 2\pi - 0.3398 + 2n\pi \approx 5.9433 + 2n\pi$$